Post-quantum key exchange for the TLS protocol from R-LWE

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joint work with
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This work: R-LWE in TLS

- All (public-key) ciphersuites currently offered in TLS will break if a large-scale quantum computer is built

- **This work:** build ciphersuites that (hopefully) won't

e.g.

RLWE-ECDSA-AES128-GCM-SHA256

(openssl.org)

TLS v1.2 cipher suites

```
TLS_RSA_WITH_NULL_SHA256

TLS_RSA_WITH_AES_128_CBC_SHA256
TLS_RSA_WITH_AES_256_CBC_SHA256
TLS_RSA_WITH_AES_128_GCM_SHA256
TLS_RSA_WITH_AES_256_GCM_SHA384

TLS_DH_RSA_WITH_AES_128_CBC_SHA256
TLS_DH_RSA_WITH_AES_256_CBC_SHA256
TLS_DH_RSA_WITH_AES_128_GCM_SHA256
TLS_DH_RSA_WITH_AES_256_GCM_SHA384

TLS_DH_DSS_WITH_AES_128_CBC_SHA256
TLS_DH_DSS_WITH_AES_256_CBC_SHA256
TLS_DH_DSS_WITH_AES_128_GCM_SHA256
TLS_DH_DSS_WITH_AES_256_GCM_SHA384

TLS_DHE_RSA_WITH_AES_128_CBC_SHA256
TLS_DHE_RSA_WITH_AES_256_CBC_SHA256
TLS_DHE_RSA_WITH_AES_128_GCM_SHA256
TLS_DHE_RSA_WITH_AES_256_GCM_SHA384

TLS_DHE_DSS_WITH_AES_128_CBC_SHA256
TLS_DHE_DSS_WITH_AES_256_CBC_SHA256
TLS_DHE_DSS_WITH_AES_128_GCM_SHA256
TLS_DHE_DSS_WITH_AES_256_GCM_SHA384

TLS_ECDH_RSA_WITH_AES_128_CBC_SHA256
TLS_ECDH_RSA_WITH_AES_256_CBC_SHA384
TLS_ECDH_RSA_WITH_AES_128_GCM_SHA256
TLS_ECDH_RSA_WITH_AES_256_GCM_SHA384

TLS_ECDH_ECDSA_WITH_AES_128_CBC_SHA256
TLS_ECDH_ECDSA_WITH_AES_256_CBC_SHA384
TLS_ECDH_ECDSA_WITH_AES_128_GCM_SHA256
TLS_ECDH_ECDSA_WITH_AES_256_GCM_SHA384

TLS_ECDHE_RSA_WITH_AES_128_CBC_SHA256
TLS_ECDHE_RSA_WITH_AES_256_CBC_SHA384
TLS_ECDHE_RSA_WITH_AES_128_GCM_SHA256
TLS_ECDHE_RSA_WITH_AES_256_GCM_SHA384

TLS_ECDHE_ECDSA_WITH_AES_128_CBC_SHA256
TLS_ECDHE_ECDSA_WITH_AES_256_CBC_SHA384
TLS_ECDHE_ECDSA_WITH_AES_128_GCM_SHA256
TLS_ECDHE_ECDSA_WITH_AES_256_GCM_SHA384

TLS_DH_anon_WITH_AES_128_CBC_SHA256
TLS_DH_anon_WITH_AES_256_CBC_SHA256
TLS_DH_anon_WITH_AES_128_GCM_SHA256
TLS_DH_anon_WITH_AES_256_GCM_SHA384
```

This work: R-LWE key agreement in TLS

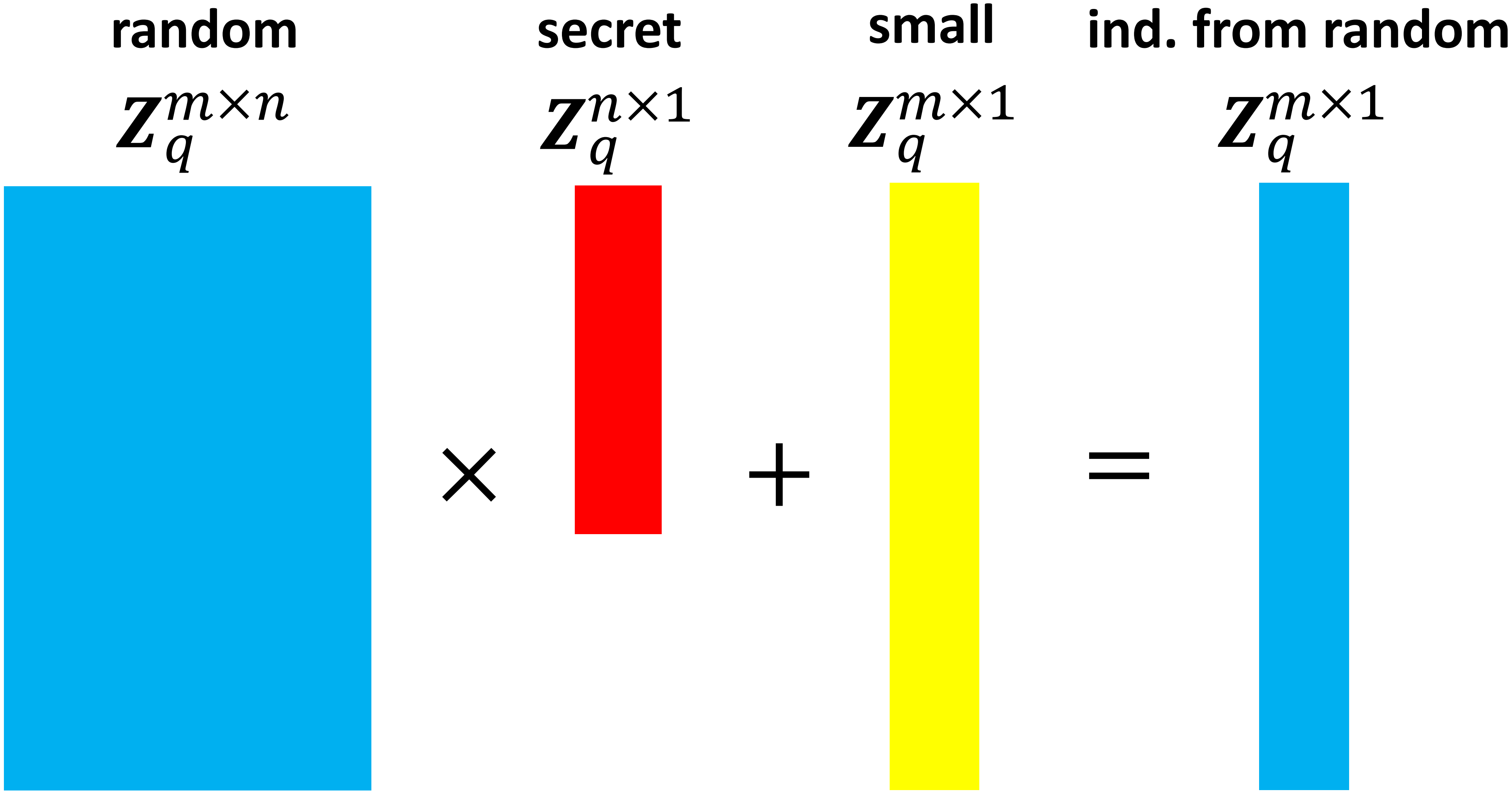
- In this work, we start by looking at **post-quantum key agreement only**

- **Assumption:** *large-scale quantum computers don't exist now, but what if we want to protect today's communications against tomorrow's adversary?*

- Signatures still done with traditional primitives RSA/ECDSA (we only need authentication to be secure now)

e.g. `RLWE-ECDSA-AES128-GCM-SHA256`

The learning with errors (LWE) problem



LWE problem: given **blue**, find **red**

The learning with errors (LWE) problem

$$\overset{\substack{\textbf{random}\\ \boldsymbol{Z}_{13}^{7\times 4}}}{\begin{bmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{bmatrix}} \times \overset{\substack{\textbf{secret}\\ \boldsymbol{Z}_{13}^{4\times 1}}}{\begin{bmatrix} \\ \\ \\ \end{bmatrix}} + \overset{\substack{\textbf{small}\\ \boldsymbol{Z}_{13}^{7\times 1}}}{\begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}} = \overset{\substack{\textbf{ind. from random}\\ \boldsymbol{Z}_{13}^{7\times 1}}}{\begin{bmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{bmatrix}}$$

LWE problem: given **blue**, find **red**

The learning with errors (LWE) problem

$$
\overset{\textbf{random}}{\overset{\mathbb{Z}_{13}^{7\times4}}{\begin{bmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{bmatrix}}}
\times
\overset{\textbf{secret}}{\overset{\mathbb{Z}_{13}^{4\times1}}{\begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix}}}
+
\overset{\textbf{small}}{\overset{\mathbb{Z}_{13}^{7\times1}}{\begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}}}
=
\overset{\textbf{ind. from random}}{\overset{\mathbb{Z}_{13}^{7\times1}}{\begin{bmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{bmatrix}}}
$$

LWE problem: given **blue**, find **red**

Toy example versus real-world example

$\boldsymbol{Z}_{13}^{7\times 4}$

4	1	11	10
5	5	9	5
3	9	0	10
1	3	3	2
12	7	3	4
6	5	11	4
3	3	5	0

$\boldsymbol{Z}_{4093}^{640\times 256}$

$$640\left\{\begin{array}{ccccc} \multicolumn{4}{c}{\overbrace{\hspace{8em}}^{256}} & \\ 2738 & 3842 & 3345 & 2979 & \cdots \\ 2896 & 595 & 3607 & & \\ 377 & 1575 & & & \\ 2760 & & & & \\ \vdots & & & & \end{array}\right.$$

640 × 256 × 12 = 1966080 bits

= 245 kB !!

The learning with errors (LWE) problem

$$\begin{array}{ccccccc}
\textbf{random} & & \textbf{secret} & & \textbf{small} & & \textbf{ind. from random} \\
\boldsymbol{Z}_{13}^{7\times 4} & & \boldsymbol{Z}_{13}^{4\times 1} & & \boldsymbol{Z}_{13}^{7\times 1} & & \boldsymbol{Z}_{13}^{7\times 1} \\
\begin{bmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{bmatrix} & \times & \begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix} & + & \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} & = & \begin{bmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{bmatrix}
\end{array}$$

LWE problem: given blue, find red

The ring learning with errors (R-LWE) problem

$$\underset{\substack{\textbf{random}\\ \mathbb{Z}_{13}^{7\times 4}}}{\begin{bmatrix} 4 & 1 & 11 & 10 \\ 10 & 4 & 1 & 11 \\ 11 & 10 & 4 & 1 \\ 1 & 11 & 10 & 4 \\ 12 & 7 & 3 & 4 \\ 4 & 12 & 7 & 3 \\ 3 & 4 & 12 & 7 \end{bmatrix}} \times \underset{\substack{\textbf{secret}\\ \mathbb{Z}_{13}^{4\times 1}}}{\begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix}} + \underset{\substack{\textbf{small}\\ \mathbb{Z}_{13}^{7\times 1}}}{\begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}} = \underset{\substack{\textbf{ind. from random}\\ \mathbb{Z}_{13}^{7\times 1}}}{\begin{bmatrix} 4 \\ 6 \\ 4 \\ 0 \\ 5 \\ 8 \\ 2 \end{bmatrix}}$$

The ring learning with errors (R-LWE) problem

$$\underset{\mathbb{Z}_{13}^{7\times 4}}{\overset{\textbf{random}}{\begin{bmatrix} 4 & 1 & 11 & 10 \\ 3 & 4 & 1 & 11 \\ 2 & 3 & 4 & 1 \\ 12 & 2 & 3 & 4 \\ 12 & 7 & 3 & 4 \\ 9 & 12 & 7 & 3 \\ 10 & 9 & 12 & 7 \end{bmatrix}}} \times \underset{\mathbb{Z}_{13}^{4\times 1}}{\overset{\textbf{secret}}{\begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix}}} + \underset{\mathbb{Z}_{13}^{7\times 1}}{\overset{\textbf{small}}{\begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}}} = \underset{\mathbb{Z}_{13}^{7\times 1}}{\overset{\textbf{ind. from random}}{\begin{bmatrix} 4 \\ 3 \\ 4 \\ 12 \\ 5 \\ 12 \\ 11 \end{bmatrix}}}$$

The ring learning with errors (R-LWE) problem

$$\underset{\mathbb{Z}_{13}^{4\times 4}}{\textbf{random}} \qquad \underset{\mathbb{Z}_{13}^{4\times 1}}{\textbf{secret}} \qquad \underset{\mathbb{Z}_{13}^{4\times 1}}{\textbf{small}} \qquad \underset{\mathbb{Z}_{13}^{4\times 1}}{\textbf{ind. from random}}$$

$$\begin{bmatrix} 4 & 1 & 11 & 10 \\ 3 & 4 & 1 & 11 \\ 2 & 3 & 4 & 1 \\ 12 & 2 & 3 & 4 \end{bmatrix} \times \begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 4 \\ 12 \end{bmatrix}$$

LWE problem: given blue, find red

The ring learning with errors (R-LWE) problem

$$\boldsymbol{Z}_{13}^{4\times 4} \longrightarrow \boldsymbol{Z}_{13}[x]/(x^4+1)$$

4	1	11	10
3	4	1	11
2	3	4	1
12	2	3	4

$$\longrightarrow 4 + 1x + 11x^2 + 10x^3$$

$$\longrightarrow = x \cdot (4 + 1x + 11x^2 + 10x^3)$$

$$\longrightarrow = x^2 \cdot (4 + 1x + 11x^2 + 10x^3)$$

$$\longrightarrow = x^3 \cdot (4 + 1x + 11x^2 + 10x^3)$$

The ring learning with errors (R-LWE) problem

$$
\begin{array}{rl}
 & 4 + 1x + 11x^2 + 10x^3 \\
\times & 6 + 9x + 11x^2 + 11x^3 \\
+ & 0 - 1x + 1x^2 + 1x^3 \\
\hline
 & 10 + 5x + 10x^2 + 7x^3 \\
\hline
\end{array}
$$

$$\frac{\mathbb{Z}_{13}[x]}{\langle x^4 + 1 \rangle}$$

R-LWE problem: given **blue**, find **red**

The ring learning with errors (R-LWE) problem

$$
\begin{array}{rl}
 & \mathbf{4 + 1x + 11x^2 + 10x^3} \\
\times & \mathbf{1 + 0x - 1x^2 - 1x^3} \\
+ & \mathbf{0 - 1x + 1x^2 + 1x^3} \\
\hline
 & \mathbf{3 + 8x + 5x^2 + 6x^3} \\
\hline
\end{array}
\qquad
\frac{\boldsymbol{Z}_{13}[x]}{\langle x^4 + 1 \rangle}
$$

R-LWE problem (small secrets): given **blue**, find (small!) **red**

The ring learning with errors (R-LWE) problem
(the 128-bit secure version)

$$\begin{array}{rl} & \mathbf{2792930407} + \cdots + \mathbf{2938465015}\boldsymbol{x}^{\mathbf{1023}} \\ \times & \mathbf{5} - \mathbf{3}\,\boldsymbol{x} \ldots + \mathbf{9}\boldsymbol{x}^{\mathbf{1022}} - \mathbf{1}\boldsymbol{x}^{\mathbf{1023}} \\ + & \mathbf{2} + \mathbf{4}\,\boldsymbol{x} \ldots - \mathbf{0}\boldsymbol{x}^{\mathbf{1022}} + \mathbf{6}\boldsymbol{x}^{\mathbf{1023}} \\ \hline & \mathbf{3159804584} + \cdots + \mathbf{1153769078}\boldsymbol{x}^{\mathbf{1023}} \\ \hline \end{array}$$

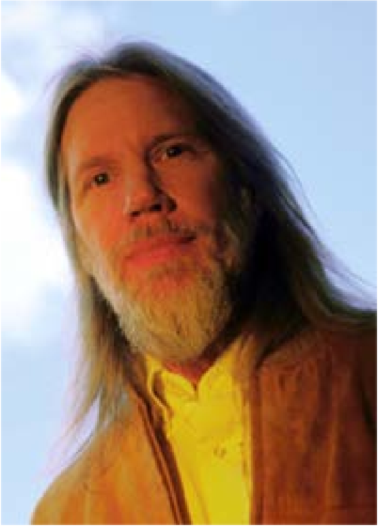
$$\frac{\boldsymbol{Z}_{2^{32}-1}[x]}{\langle x^{1024} + 1 \rangle}$$

R-LWE problem: given **blue**, find (small!) **red**

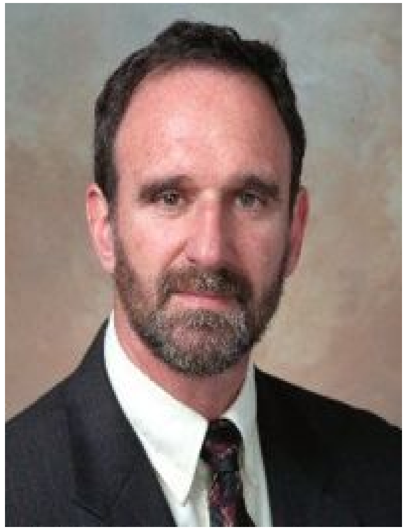
R-LWE-DH: key agreement in $R_q = \boldsymbol{Z}_q[x]/\langle x^n + 1\rangle$

public: “big” a $\in R_q$

secret: “small” $e, s \in R_q$



secret: “small” $e', s' \in R_q$



$$a \cdot s + e \longrightarrow$$

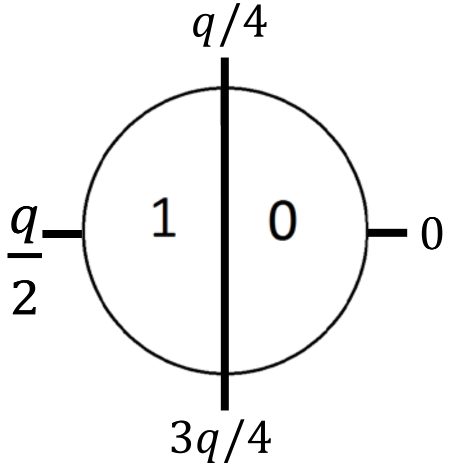
$$\longleftarrow a \cdot s' + e'$$

$$\big(s \cdot (a \cdot s' + e')\big) \approx s \cdot a \cdot s'$$

$$\big(s' \cdot (a \cdot s + e)\big) \approx s \cdot a \cdot s'$$

Approximate agreement mod q

the usual **ROUND** function



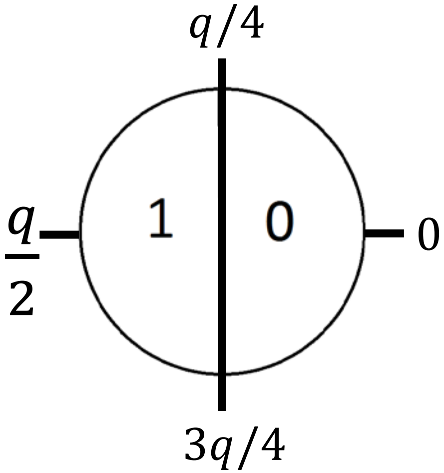
$$4079331841 + 1894732145 \cdot x + \cdots + 472608255 \cdot x^{1022} + 516748383 \cdot x^{1023}$$

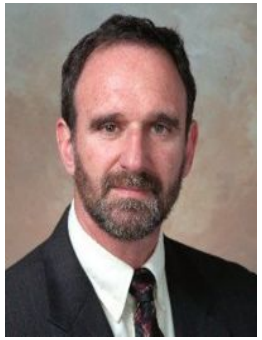
$$4079332556 + 1894733033 \cdot x + \cdots + 472607765 \cdot x^{1022} + 516748363 \cdot x^{1023}$$

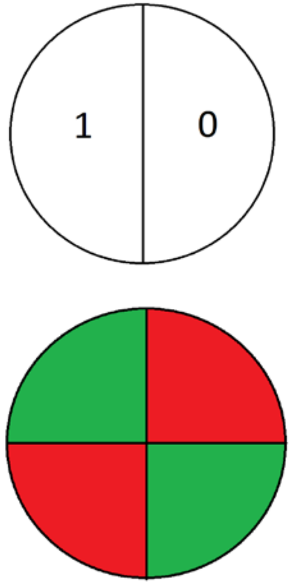
ROUND $\quad 0 \quad 1 \quad 0 \quad 0$

This will work most of the time (fails $\approx 1/2^{10}$), but we need **exact agreement**
i.e. what happens if one of the coefficients is in the **"danger zone(s)"**

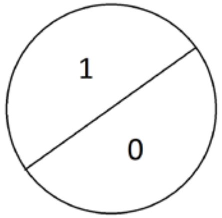
Making approximate agreement exact in $\boldsymbol{Z}_q$

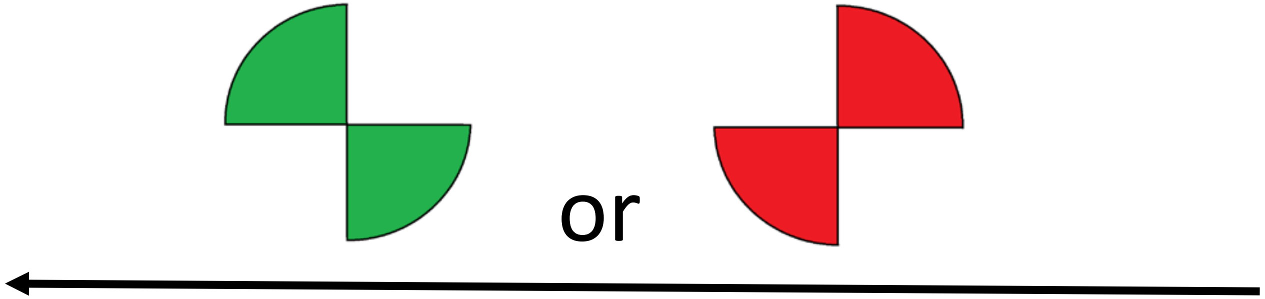




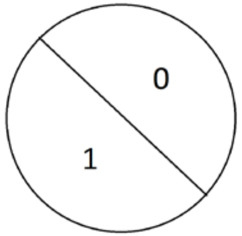


if 





else 

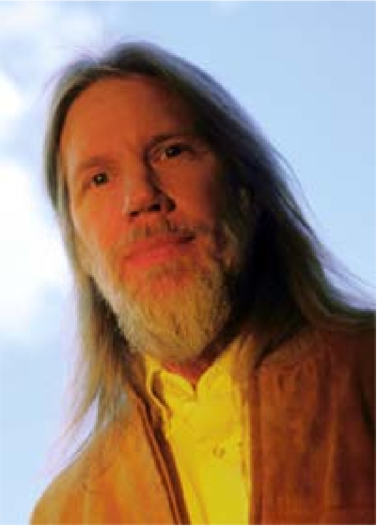


(Peikert's reconciliation mechanisms: http://eprint.iacr.org/2014/070.pdf)
two values $u, v \in \boldsymbol{Z}_q$ will agree so long as $|u - v| < q/8$ (i.e. **always!**)

R-LWE-DH: exact key agreement

public: “big” a $\in R_q$

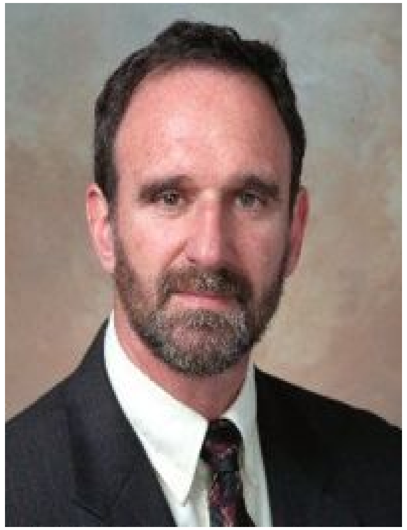
secret: “small” $e, s \in R_q$



$a \cdot s + e$

$a \cdot s' + e'$ and $\{\ ,\ \}^n \in \{0,1\}^n$

secret: “small” $e', s' \in R_q$



RECONCILE$(s \cdot (a \cdot s' + e'), \{\ ,\ \}^n)$ = ROUND$\left(s' \cdot (a \cdot s + e)\right)$

both parties now share $k \in \{0,1\}^n$

Security aspects

A secure key agreement protocol

- Prove that scheme is secure under “Exact DDH-like problem”
- Show that “Exact DDH-like problem” is hard if decision R-LWE problem is

Secure integration into the TLS

- Integrate R-LWE key agreement into the TLS protocol
- Use Jager *et al.* “Authenticated and confidential channel establishment” (ACCE) model (Crypto2012)
- Prove that “TLS-signed R-LWE is a secure ACCE”

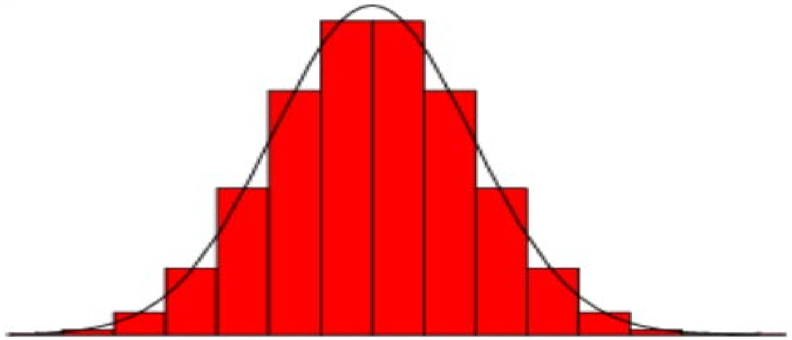
Implementation aspect 1: polynomial arithmetic

- Polynomial multiplication in $R_q = \boldsymbol{Z}_q[x]/\langle x^{1024} + 1\rangle$ done with Nussbaumer's FFT ($2^l = r \cdot k$)

$$\frac{R[X]}{\langle X^{2^l} + 1\rangle} \cong \frac{\left(\frac{R[Z]}{\langle Z^r + 1\rangle}\right)[X]}{\langle X^k - Z\rangle}$$

- Rather than working modulo degree-1024 polynomial with coefficients in $\boldsymbol{Z}_q$, work modulo:-
  - degree-256 polynomial whose coefficients are themselves polynomials modulo a degree-4 polynomial, or
  - degree-32 polynomials whose coefficients are polynomials modulo degree-8 polynomials whose coefficients are polynomials …

Implementation aspect 2: sampling discrete Gaussians



$$D_{\mathbf{Z},\sigma}(x) = \frac{1}{S}\, e^{-\frac{x^2}{2\sigma^2}} \quad \text{for } x \in \mathbf{Z}$$

(for us: $\sigma \approx 3.2\,,\, S = 8$)

- Security (proofs) require "small" elements to be within statistical distance 2^{-128} of *true* discrete Gaussian $D_{Z,\sigma}(x)$

- **Inversion sampling:** precompute table of cumulative probabilities
  (for us: 52 elements of 192-bits in size: ≈ 10,000 bits)

- Each coefficient requires six 192-bit integer comparisons (51 if "constant-time"), and there are 1024 coefficients!!!

The price of post-quantum paranoia, part I

Table 1: Average cycle count of standalone cryptographic operations (on client computer)

Operation	Cycles	
	constant-time	**non-constant-time**
sample $\xleftarrow{\$} \chi$	1 042 700	668 000
FFT multiplication	342 800	—
FFT addition	1 660	—
dbl$(\cdot)$ and crossrounding $\langle\cdot\rangle_{2q,2}$	23 500	21 300
rounding $\lfloor\cdot\rceil_{2q,2}$	5 500	3,700
reconciliation rec$(\cdot,\cdot)$	14 400	6 800

(Intel Core i5 (4570R) @ 2.7GHz)

The price of post-quantum paranoia, part II

Table 2: Average runtime in milliseconds of cryptographic operations using `openssl speed`

Operation	Client constant-time	Server constant-time	Client non-constant-time	Server non-constant-time
R-LWE key generation	0.9	1.7	0.6	1.3
R-LWE Bob shared secret	0.5	(1.1)	0.4	(0.9)
R-LWE Alice shared secret	(0.1)	0.4	(0.1)	0.4
Total R-LWE runtime	1.4	2.1	1.0	1.7
EC point multiplication, `nistp256`	0.4	0.7	—	—
Total ECDH runtime	0.8	1.4	—	—
RSA sign, 3072-bit key	(3.7)	8.8	—	—
RSA verify, 3072-bit key	0.1	(0.2)	—	—

Numbers in parentheses are reported for completeness, but do not contribute to the runtime in the client and server's role in the TLS protocol.

The price of post-quantum paranoia, part III

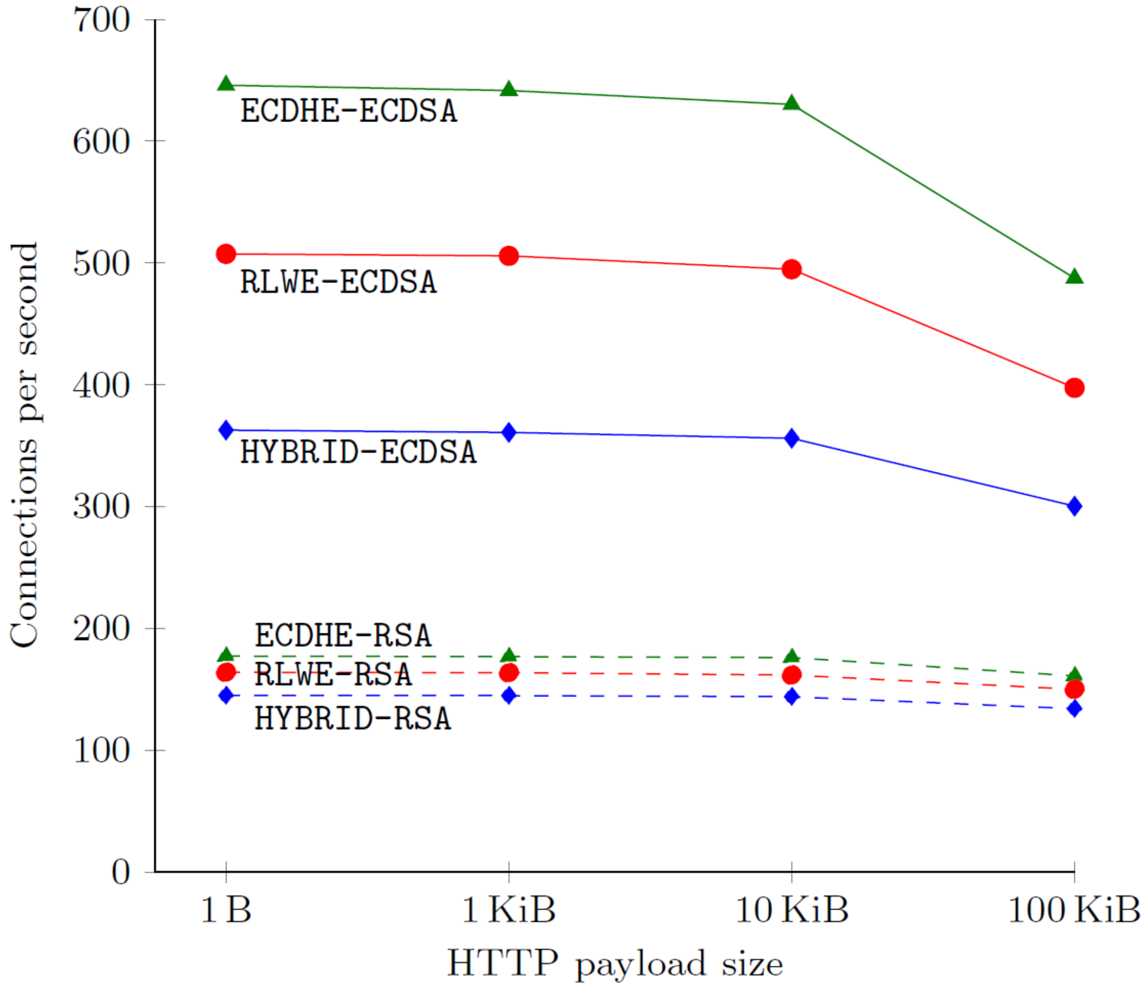


Table 3: Performance of HTTPS using Apache with OpenSSL

	ECDHE		RLWE		HYBRID	
	ECDSA	RSA	ECDSA	RSA	ECDSA	RSA
Connections / second						
— 1 B payload	645.9	177.4	507.5	164.2	362.9	145.1
— 1 KiB payload	641.6	177.0	505.9	163.8	361.0	145.0
— 10 KiB payload	630.2	176.2	494.9	161.9	356.2	144.1
— 100 KiB payload	487.6	161.2	397.6	150.2	300.5	134.3
Connection time (ms)	6.0	14.0	45.6	54.0	47.2	54.6
Handshake (bytes)	1 278	2 360	9 469	10 479	9 607	10 690

Summary and future work

- If you want to protect today's secrets against tomorrow's quantum adversary, use

  RLWE-ECDSA-AES128-GCM-SHA256

  in TLS for a small overhead

- **Future work, part II:** protecting tomorrow's secrets too!

  RLWE-RLWE-AES128-GCM-SHA256

  LWE-LWE-AES128-GCM-SHA256

  ????-????-AES128-GCM-SHA256

- **Future work, part I:** a tonne of unexplored optimizations (this is our first go)
  - e.g: we didn't do assembly/precomputation/parallelizing
  - e.g: alternative FFT's
  - e.g: much faster/compact sampling algorithms likely

The paper (to appear at Oakland S&P)
http://eprint.iacr.org/2014/599.pdf

Magma code:
http://research.microsoft.com/en-US/downloads/6bd592d7-cf8a-4445-b736-1fc39885dc6e/default.aspx

C code integrated into OpenSSL:
https://github.com/dstebila/rlwekex